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→ Evaluation of the Integral

$$\int_0^{\infty} v^n \exp\left[-(v-u)^2 - \frac{x}{v}\right] dv$$

→ M. T. Chahine and NASA CR 52018;
R. Narasimha

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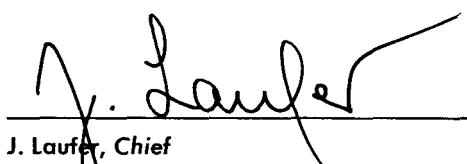
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August 5, 1963

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PREFACE

Dr. R. Narasimha was a research fellow at the Graduate Aeronautical Laboratories, California Institute of Technology, Pasadena, California, and is now at the Indian Institute of Science, Bangalore 12, India.

ABSTRACT

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The function defined by the integral

$$g_n(x, u) = \int_0^\infty v^n \exp \left[- (v - u)^2 - \frac{x}{v} \right] dv$$

is tabulated to six decimals in the ranges $0 \leq x \leq 20$ and $-2.5 \leq u \leq 10$ for $n = 0, 1, 2$, and asymptotic expressions for the function are given for limiting values of x and u .

AUTHOR

I. INTRODUCTION

In various problems of theoretical physics there arise certain functions defined by the definite integral

$$g_n(x, u) = \int_0^\infty v^n \exp \left[- (v - u)^2 - \frac{x}{v} \right] dv \quad (1)$$

where x and u are two real numbers, and x is always positive. In general these functions occur in dealing with the absorption and scattering of particles having a Maxwellian distribution of velocities, as for instance in computing the absorption coefficient for a beam of thermal neutrons (Refs. 1,2). They also occur in the

kinetic theory of gases where a simple relaxation model is used for the collision terms in the Boltzmann equation (Refs. 3, 4, 5); the moments of the distribution functions can then often be expressed in terms of the functions g_n .

The case $u = 0$ has been studied by Abramowitz (Ref. 6) who has given, for the function $g_0(x, 0)$, a series expansion suitable for small values of x and an asymptotic expansion for large values of x .

The function $g_n(x, u)$ is not, in general, reducible to any of the standard tabulated integrals. In this report we first discuss the general properties of the functions and their asymptotic behavior; then we present tables of their values as found by numerical integration on a computer.

II. GENERAL PROPERTIES

The function $g_n(x, u)$ satisfies the following differential equation and recurrence relations:

$$-x \frac{\partial^3 g_n}{\partial x^3} + (n - 1) \frac{\partial^2 g_n}{\partial x^2} - 2u \frac{\partial g_n}{\partial x} - 2g_n = 0 \quad (2a)$$

$$\frac{\partial g_n}{\partial x} = -g_{n-1} \quad (2b)$$

$$2g_n = 2ug_{n-1} + (n - 1)g_{n-2} + xg_{n-3} \quad (2c)$$

Also, the function $g_n(x, u)$ is finite for $x > 0$ for all values of u and n , but infinite at $x = 0$ for $n \leq -1$.

Special values of interest are

$$g_0(0, u) = \frac{\pi^{1/2}}{2} [1 + \operatorname{erf}(u)] \quad (3a)$$

$$g_1(0, u) = \frac{\exp(-u^2)}{2} + \frac{\pi^{1/2}}{2} u [1 + \operatorname{erf}(u)] \quad (3b)$$

$$g_2(0, u) = u \frac{\exp(-u^2)}{2} + \frac{\pi^{1/2}}{2} \left(\frac{1}{2} + u^2 \right) [1 + \operatorname{erf}(u)] \quad (3c)$$

III. EXPANSIONS FOR LIMITING VALUES OF u A. Large Positive u

The asymptotic behavior of the function for large u can be obtained by the saddle point method (Ref. 7). In applying this method it is convenient to write

$$V = \frac{v}{u} , \quad X = \frac{x}{u^3} \quad (4)$$

When such a change is made, Eq. (1) becomes

$$g_n(x, u) = u^{n+1} \int_0^\infty V^n \exp \left\{ -u^2 \left[(V - 1)^2 + \frac{X}{V} \right] \right\} dV$$

For large u most of the contribution to the integral comes from the neighborhood of the saddle point, where the argument of the exponential

$$-u^2 h(X, V) = -u^2 \left[(V - 1)^2 + \frac{X}{V} \right] \quad (5)$$

is a minimum. This point will be given by the roots of the cubic

$$V^2 \frac{\partial h(X, V)}{\partial V} = 2V^2(V - 1) - X = 0 \quad (6)$$

As $X > 0$, the saddle point in which we are interested is the only real root $V = V_0 > 1$ of Eq. (6) and is given by

$$V_0(X) = \left(\frac{X}{4} \right)^{1/3} \left\{ \left[1 + \frac{4}{27X} + \left(1 + \frac{8}{27X} \right)^{1/2} \right]^{1/3} + \left[1 + \frac{4}{27X} - \left(1 + \frac{8}{27X} \right)^{1/2} \right]^{1/3} \right\} + \frac{1}{3} \quad (7)$$

Expanding $h(X, V)$ around V_0 as usual, we obtain the asymptotic expression

$$g_n(x, u) \approx \left(\frac{\pi}{3V_0 - 2} \right)^{1/2} u^n V_0^{n+1/2} \exp [-u^2(V_0 - 1)(3V_0 - 1)] \quad (8)$$

To see the form of Eq. (8) more clearly, suppose X is small; then V_0 can be expanded in the series

$$V_0 \approx 1 + \frac{1}{2} X - \frac{1}{2} X^2 + \frac{7}{8} X^3 + \dots$$

and

$$g_n(x, u) \approx \pi^{1/2} u^n \left\{ 1 + \frac{n-1}{2} \frac{x}{u^3} + \left[\frac{n}{2} \left(\frac{n}{4} - 1 \right) + \frac{21}{16} \right] \left(\frac{x}{u^3} \right)^2 + \dots \right\} \exp \left(-\frac{x}{u} \right) \quad (9)$$

For large X , on the other hand,

$$V_0 \approx \left(\frac{X}{2} \right)^{1/3} \left(1 + \frac{4}{81X} + \dots \right) + \frac{1}{3}$$

and

$$g_n(x, u) \approx \left(\frac{\pi}{3} \right)^{1/2} \left(\frac{x}{2} \right)^{n/3} \left[1 + \frac{u}{6} \left(\frac{x}{2} \right)^{-1/3} + \frac{u^2}{24} \left(\frac{x}{2} \right)^{-2/3} + \dots \right] \exp \left[-3 \left(\frac{x}{2} \right)^{2/3} \right] \quad (10)$$

Note that the first term in Eq. (10) is independent of u , and is identical with the first term of the expansion for $g_n(x, 0)$ as given by Eq. (12).

Table 1 shows a comparison, for $u = 5$, of the exact values of the functions g_0 and g_1 with the asymptotic expression for small x , computed for the same number of terms given in Eq. (9).

B. Small Positive u

For small u one can expand the integrand in Eq. (1) and obtain

$$g_n(x, u) = \int_0^\infty v^n \left[\sum_{k=0}^{\infty} \frac{u^k (2v - u)^k}{k!} \right] \exp \left(-v^2 - \frac{x}{v} \right) dv \\ = (1 - u^2) g_n(x, 0) + 2u g_{n+1}(x, 0) + 2u^2 g_{n+2}(x, 0) + O(u^3) \quad (11)$$

We can now obtain expansions for small or large x using the known properties of the functions $g_n(x, 0)$ (Ref. 6). Thus, for large x , Abramowitz gives

$$g_n(x, 0) \approx \left(\frac{\pi}{3} \right)^{1/2} \left(\frac{x}{2} \right)^{n/3} \left\{ a_0 + \frac{a_1(n)}{3 \left(\frac{x}{2} \right)^{2/3}} + \frac{a_2(n)}{\left[3 \left(\frac{x}{2} \right)^{2/3} \right]^2} + \frac{a_3(n)}{\left[3 \left(\frac{x}{2} \right)^{2/3} \right]^3} + \dots \right\} \\ \times \exp \left[-3 \left(\frac{x}{2} \right)^{2/3} \right] \quad (12)$$

The values of a_0 and a_n are

$$a_0 = 1 \quad a_1(n) = \frac{1}{12} (3n^2 + 3n - 1)$$

and the recurrence relation for $a_i(n)$ is given by

$$12(i+2) a_{i+2} = -(12i^2 + 36i - 3n^2 - 3n + 25) a_{i+1} + \frac{1}{2} (n-2i)(3i+3-n)(3i+3+2n) a_i$$

where $i = 0, 1, 2, \dots$.

Table 2 compares the values from the asymptotic expression, computed by using a_0, a_1, a_2 and a_3 in Eq. (12), with the exact solutions for $u = 0.2$. The asymptotic solutions in Tables 1 and 2 were evaluated on a desk computer using five-place logarithm tables.

C. Negative u

The asymptotic behavior of $g_n(x, u)$ for $u < 0$ can be worked out along similar lines, but no detailed results will be quoted here as the roots of the counterpart to Eq. (6) need rather complicated classification in terms of the values of x . However, we may note the following general results. For small negative u Eq. (11) is still valid with due regard to the sign of u . Also for sufficiently large x , as noted earlier, the asymptotic behavior of g_n is, to a first approximation, independent of u , so the result will be true for negative u also. Finally, for large negative u the values of g_n are too small to be of any particular interest.

IV. METHOD OF NUMERICAL COMPUTATION

The numerical computation presented in Table 3 for $n = 0, 1$, and 2 in the range $0 \leq x \leq 20$ and $-2.5 \leq u \leq 10$ were carried out on the IBM 7090 computer by applying a step-by-step integration with $\Delta v = 0.001$ using a second order Gaussian integration formula in each interval. The computations are accurate to 8 figures; however only the first 6 decimals are presented. Extension of the tables for other values of n is easily performed using the recurrence relation in Eq. (2c).

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Table 1. A comparison between the numerical solution and the asymptotic solution of Eq. (1) for large u

x	$g_0(x, + 5)$		$g_1(x, + 5)$	
	Numerical solution	Asymptotic solution	Numerical solution	Asymptotic solution
0.00	1.772453	1.772454	8.862266	8.862265
0.05	1.754445	1.754467	8.774095	8.774087
0.10	1.736623	1.736663	8.686819	8.686788
0.15	1.718987	1.719041	8.600430	8.600361
0.20	1.701534	1.701591	8.514917	8.514779
0.25	1.684262	1.684324	8.430273	8.430075
0.50	1.600553	1.600605	8.019763	8.019026
1.00	1.445665	1.445480	7.258880	7.256246
1.50	1.306068	1.305435	6.571550	6.566212
2.50	1.066739	1.064896	5.389269	5.377254
5.00	0.645580	0.640377	3.293682	3.265132
10.00	0.240068	0.232295	1.247305	1.206571
15.00	0.090923	0.084618	0.480326	0.471821
20.00	0.035006	0.030922	0.187789	0.168408

Table 2. A comparison between the numerical solution and the asymptotic solution of Eq. (1) for small u

x	$g_0(x, +0.2)$	
	Numerical solution	Asymptotic solution
3	0.032085	0.031901
4	0.014607	0.014483
5	0.007064	0.006984
6	0.003574	0.003524
7	0.001873	0.001842
8	0.001011	0.000991
9	0.000559	0.000546
10	0.000315	0.000307
15	0.000023	0.000022
20	0.000002	0.000002

Table 3. Numerical values of $g_0(x, u)$, $g_1(x, u)$ and $g_2(x, u)$

$$g_n(x, u) = \int_0^{+\infty} v^n \exp \left[-(v - u)^2 - \frac{x}{v} \right] dv$$

$x = 0.0$ (0.01) 0.1 (0.05) 0.5 (0.1) 1.0 (0.5) 5.0 (1.0) 10.0 (5.0) 20

$u = -2.5$ (0.5) -1.0 (0.2) $+1.0$ (0.5) 2.5 (2.5) 5.0 (5.0) 10.0

Five D, except for $g_2(x < 6.0, 10.0)$ where the accuracy is 4 D.

(Results truncated after the sixth decimal.)

Table 3 (Cont'd)

 $u = -2.50$

x	δ_0	δ_1	δ_2
0.	0.000361	0.000064	0.000021
0.010000	0.000304	0.000060	0.000021
0.020000	0.000272	0.000057	0.000020
0.030000	0.000247	0.000055	0.000020
0.040000	0.000227	0.000052	0.000019
0.050000	0.000210	0.000050	0.000019
0.060000	0.000195	0.000048	0.000018
0.070000	0.000183	0.000046	0.000018
0.080000	0.000171	0.000045	0.000017
0.090000	0.000161	0.000043	0.000017
0.100000	0.000152	0.000041	0.000016
0.150000	0.000116	0.000035	0.000014
0.200000	0.000092	0.000030	0.000013
0.250000	0.000075	0.000025	0.000011
0.300000	0.000062	0.000022	0.000010
0.350000	0.000051	0.000019	0.000009
0.400000	0.000043	0.000017	0.000008
0.450000	0.000037	0.000015	0.000007
0.500000	0.000032	0.000013	0.000007
0.600000	0.000024	0.000010	0.000006
0.700000	0.000018	0.000008	0.000005
0.800000	0.000014	0.000007	0.000004
0.900000	0.000011	0.000006	0.000003
1.000000	0.000009	0.000005	0.000003
1.500000	0.000003	0.000002	0.000001
2.000000	0.000001	0.000001	0.000001
2.500000	0.000001	0.000000	0.000000
3.000000	0.000000	0.000000	0.000000
3.500000	0.000000	0.000000	0.000000
4.000000	0.000000	0.000000	0.000000
4.500000	0.000000	0.000000	0.000000
5.000000	0.000000	0.000000	0.000000
6.000000	0.000000	0.000000	0.000000
7.000000	0.000000	0.000000	0.000000
8.000000	0.000000	0.000000	0.000000
9.000000	0.000000	0.000000	0.000000
10.000000	0.000000	0.000000	0.000000
15.000000	0.000000	0.000000	0.000000
20.000000	0.000000	0.000000	0.000000

Table 3 (Cont'd)

 $u = -2.0$

x	ξ_0	ξ_1	ξ_2
0.	0.004146	0.000867	0.000339
0.010000	0.003577	0.000829	0.000331
0.020000	0.003238	0.000795	0.000323
0.030000	0.002976	0.000764	0.000315
0.040000	0.002760	0.000735	0.000307
0.050000	0.002576	0.000708	0.000300
0.060000	0.002415	0.000683	0.000293
0.070000	0.002272	0.000660	0.000287
0.080000	0.002145	0.000638	0.000280
0.090000	0.002030	0.000617	0.000274
0.100000	0.001926	0.000597	0.000268
0.150000	0.001516	0.000512	0.000240
0.200000	0.001229	0.000444	0.000216
0.250000	0.001016	0.000388	0.000195
0.300000	0.000853	0.000341	0.000177
0.350000	0.000723	0.000302	0.000161
0.400000	0.000620	0.000268	0.000147
0.450000	0.000535	0.000240	0.000134
0.500000	0.000465	0.000215	0.000123
0.600000	0.000356	0.000174	0.000104
0.700000	0.000278	0.000142	0.000088
0.800000	0.000220	0.000118	0.000075
0.900000	0.000176	0.000098	0.000064
1.000000	0.000143	0.000082	0.000055
1.500000	0.000055	0.000036	0.000027
2.000000	0.000024	0.000018	0.000014
2.500000	0.000012	0.000009	0.000008
3.000000	0.000006	0.000005	0.000005
3.500000	0.000003	0.000003	0.000003
4.000000	0.000002	0.000002	0.000002
4.500000	0.000001	0.000001	0.000001
5.000000	0.000001	0.000001	0.000001
6.000000	0.000000	0.000000	0.000000
7.000000	0.000000	0.000000	0.000000
8.000000	0.000000	0.000000	0.000000
9.000000	0.000000	0.000000	0.000000
10.000000	0.000000	0.000000	0.000000
15.000000	0.000000	0.000000	0.000000
20.000000	0.000000	0.000000	0.000000

Table 3 (Cont'd)

 $u = -1.50$

x	ε_0	ε_1	ε_2
0.	0.030039	0.007642	0.003557
0.010000	0.026519	0.007362	0.003482
0.020000	0.024347	0.007108	0.003409
0.030000	0.022634	0.006873	0.003339
0.040000	0.021200	0.006654	0.003272
0.050000	0.019961	0.006448	0.003206
0.060000	0.018869	0.006254	0.003143
0.070000	0.017893	0.006071	0.003081
0.080000	0.017011	0.005896	0.003021
0.090000	0.016209	0.005730	0.002963
0.100000	0.015473	0.005572	0.002907
0.150000	0.012530	0.004876	0.002646
0.200000	0.010404	0.004305	0.002417
0.250000	0.008787	0.003827	0.002214
0.300000	0.007515	0.003421	0.002033
0.350000	0.006490	0.003072	0.001871
0.400000	0.005650	0.002769	0.001725
0.450000	0.004951	0.002504	0.001593
0.500000	0.004364	0.002272	0.001474
0.600000	0.003438	0.001884	0.001267
0.700000	0.002750	0.001576	0.001095
0.800000	0.002228	0.001329	0.000950
0.900000	0.001823	0.001127	0.000827
1.000000	0.001505	0.000961	0.000723
1.500000	0.000636	0.000463	0.000385
2.000000	0.000301	0.000240	0.000216
2.500000	0.000153	0.000132	0.000126
3.000000	0.000082	0.000075	0.000076
3.500000	0.000046	0.000044	0.000047
4.000000	0.000026	0.000027	0.000029
4.500000	0.000016	0.000017	0.000019
5.000000	0.000009	0.000010	0.000012
6.000000	0.000004	0.000004	0.000005
7.000000	0.000002	0.000002	0.000002
8.000000	0.000001	0.000001	0.000001
9.000000	0.000000	0.000000	0.000001
10.000000	0.000000	0.000000	0.000000
15.000000	0.000000	0.000000	0.000000
20.000000	0.000000	0.000000	0.000000

Table 3 (Cont'd)

 $u = -1.00$

x	ε_0	ε_1	ε_2
0.	0.139403	0.044537	0.025164
0.010000	0.125986	0.043219	0.024726
0.020000	0.117367	0.042004	0.024300
0.030000	0.110425	0.040866	0.023885
0.040000	0.104515	0.039792	0.023482
0.050000	0.099336	0.038773	0.023089
0.060000	0.094714	0.037803	0.022707
0.070000	0.090536	0.036878	0.022333
0.080000	0.086724	0.035992	0.021969
0.090000	0.083220	0.035142	0.021613
0.100000	0.079980	0.034326	0.021266
0.150000	0.066729	0.030676	0.019644
0.200000	0.056828	0.027598	0.018189
0.250000	0.049078	0.024958	0.016877
0.300000	0.042828	0.022666	0.015687
0.350000	0.037680	0.020657	0.014605
0.400000	0.033372	0.018884	0.013618
0.450000	0.029721	0.017309	0.012714
0.500000	0.026596	0.015903	0.011884
0.600000	0.021550	0.013507	0.010418
0.700000	0.017690	0.011553	0.009168
0.800000	0.014674	0.009941	0.008096
0.900000	0.012279	0.008598	0.007171
1.000000	0.010352	0.007470	0.006369
1.500000	0.004789	0.003897	0.003641
2.000000	0.002436	0.002167	0.002173
2.500000	0.001320	0.001260	0.001340
3.000000	0.000749	0.000758	0.000847
3.500000	0.000440	0.000469	0.000546
4.000000	0.000266	0.000296	0.000359
4.500000	0.000165	0.000191	0.000239
5.000000	0.000104	0.000125	0.000161
6.000000	0.000044	0.000056	0.000076
7.000000	0.000019	0.000026	0.000037
8.000000	0.000009	0.000013	0.000019
9.000000	0.000004	0.000006	0.000010
10.000000	0.000002	0.000003	0.000005
15.000000	0.000000	0.000000	0.000000
20.000000	0.000000	0.000000	0.000000

Table 3 (Cont'd)

 $u = -0.80$

x	ξ_0	ξ_1	ξ_2
0.	0.228557	0.080801	0.049638
0.010000	0.208517	0.078628	0.048841
0.020000	0.195413	0.076611	0.048065
0.030000	0.184759	0.074712	0.047308
0.040000	0.175622	0.072911	0.046570
0.050000	0.167565	0.071196	0.045850
0.060000	0.160337	0.069557	0.045146
0.070000	0.153771	0.067987	0.044459
0.080000	0.147754	0.066480	0.043786
0.090000	0.142201	0.065030	0.043129
0.100000	0.137045	0.063634	0.042486
0.150000	0.115757	0.057341	0.039466
0.200000	0.099618	0.051974	0.036736
0.250000	0.086830	0.047324	0.034256
0.300000	0.076407	0.043252	0.031994
0.350000	0.067741	0.039654	0.029923
0.400000	0.060424	0.036455	0.028022
0.450000	0.054174	0.033594	0.026272
0.500000	0.048783	0.031023	0.024658
0.600000	0.039991	0.026604	0.021784
0.700000	0.033177	0.022960	0.019311
0.800000	0.027793	0.019921	0.017172
0.900000	0.023471	0.017366	0.015311
1.000000	0.019959	0.015200	0.013686
1.500000	0.009583	0.008194	0.008050
2.000000	0.005026	0.004684	0.004924
2.500000	0.002796	0.002790	0.003102
3.000000	0.001623	0.001715	0.002000
3.500000	0.000974	0.001081	0.001314
4.000000	0.000601	0.000696	0.000877
4.500000	0.000379	0.000456	0.000594
5.000000	0.000243	0.000303	0.000407
6.000000	0.000105	0.000139	0.000197
7.000000	0.000048	0.000067	0.000099
8.000000	0.000023	0.000033	0.000051
9.000000	0.000011	0.000017	0.000027
10.000000	0.000006	0.000009	0.000015
15.000000	0.000000	0.000000	0.000001
20.000000	0.000000	0.000000	0.000000

Table 3 (Cont'd)

 $u = -0.60$

x	g_0	g_1	g_2
0.	0.351073	0.138194	0.092620
0.010000	0.323316	0.134839	0.091255
0.020000	0.304820	0.131702	0.089923
0.030000	0.289631	0.128732	0.088621
0.040000	0.276503	0.125903	0.087348
0.050000	0.264852	0.123197	0.086102
0.060000	0.254339	0.120602	0.084883
0.070000	0.244742	0.118107	0.083690
0.080000	0.235905	0.115705	0.082521
0.090000	0.227713	0.113387	0.081375
0.100000	0.220078	0.111148	0.080253
0.150000	0.188229	0.100979	0.074956
0.200000	0.163715	0.092205	0.070132
0.250000	0.144045	0.084527	0.065718
0.300000	0.127834	0.077743	0.061664
0.350000	0.114220	0.071701	0.057931
0.400000	0.102622	0.066287	0.054484
0.450000	0.092632	0.061412	0.051293
0.500000	0.083947	0.057002	0.048335
0.600000	0.069635	0.049353	0.043029
0.700000	0.058397	0.042973	0.038422
0.800000	0.049410	0.037599	0.034401
0.900000	0.042117	0.033035	0.030875
1.000000	0.036129	0.029132	0.027772
1.500000	0.018012	0.016229	0.016803
2.000000	0.009744	0.009536	0.010533
2.500000	0.005566	0.005820	0.006780
3.000000	0.003309	0.003656	0.004458
3.500000	0.002029	0.002351	0.002982
4.000000	0.001276	0.001541	0.002025
4.500000	0.000819	0.001026	0.001393
5.000000	0.000535	0.000693	0.000968
6.000000	0.000239	0.000328	0.000482
7.000000	0.000112	0.000161	0.000248
8.000000	0.000054	0.000082	0.000131
9.000000	0.000027	0.000043	0.000071
10.000000	0.000014	0.000023	0.000039
15.000000	0.000001	0.000001	0.000003
20.000000	0.000000	0.000000	0.000000

Table 3 (Cont'd)

 $u = -0.40$

x	ε_0	ε_1	ε_2
0.	0.506574	0.223442	0.163910
0.010000	0.470885	0.218576	0.161700
0.020000	0.446619	0.213993	0.159538
0.030000	0.426476	0.209630	0.157420
0.040000	0.408922	0.205455	0.155344
0.050000	0.393236	0.201445	0.153310
0.060000	0.378995	0.197585	0.151315
0.070000	0.365926	0.193862	0.149358
0.080000	0.353832	0.190263	0.147437
0.090000	0.342571	0.186782	0.145552
0.100000	0.332031	0.183410	0.143701
0.150000	0.287588	0.167970	0.134926
0.200000	0.252837	0.154491	0.126872
0.250000	0.224581	0.142578	0.119451
0.300000	0.201024	0.131955	0.112593
0.350000	0.181036	0.122417	0.106237
0.400000	0.163848	0.113805	0.100335
0.450000	0.148914	0.105994	0.094844
0.500000	0.135827	0.098883	0.089724
0.600000	0.114025	0.086433	0.080477
0.700000	0.096673	0.075930	0.072373
0.800000	0.082624	0.066989	0.065239
0.900000	0.071094	0.059322	0.058933
1.000000	0.061527	0.052705	0.053339
1.500000	0.031865	0.030340	0.033188
2.000000	0.017784	0.018327	0.021312
2.500000	0.010436	0.011459	0.014018
3.000000	0.006353	0.007357	0.009398
3.500000	0.003981	0.004825	0.006402
4.000000	0.002553	0.003220	0.004420
4.500000	0.001669	0.002182	0.003088
5.000000	0.001109	0.001498	0.002180
6.000000	0.000511	0.000730	0.001115
7.000000	0.000246	0.000369	0.000588
8.000000	0.000123	0.000192	0.000318
9.000000	0.000063	0.000102	0.000176
10.000000	0.000033	0.000056	0.000099
15.000000	0.000002	0.000003	0.000007
20.000000	0.000000	0.000000	0.000001

Table 3 (Cont'd)

 $u = -0.20$

x	ε_0	ε_1	ε_2
0.	0.688862	0.342622	0.275906
0.010000	0.646207	0.335971	0.272514
0.020000	0.616564	0.329662	0.269186
0.030000	0.591673	0.323624	0.265920
0.040000	0.569790	0.317819	0.262713
0.050000	0.550091	0.312221	0.259563
0.060000	0.532094	0.306811	0.256468
0.070000	0.515482	0.301574	0.253426
0.080000	0.500031	0.296498	0.250436
0.090000	0.485574	0.291570	0.247495
0.100000	0.471983	0.286783	0.244604
0.150000	0.414026	0.264694	0.230829
0.200000	0.367952	0.245185	0.218091
0.250000	0.329968	0.227765	0.206276
0.300000	0.297921	0.212089	0.195286
0.350000	0.270435	0.197897	0.185042
0.400000	0.246571	0.184986	0.175475
0.450000	0.225649	0.173191	0.166525
0.500000	0.207163	0.162380	0.158139
0.600000	0.176017	0.143279	0.142882
0.700000	0.150882	0.126977	0.129390
0.800000	0.130273	0.112952	0.117411
0.900000	0.113163	0.100806	0.106737
1.000000	0.098811	0.090228	0.097198
1.500000	0.053173	0.053662	0.062165
2.000000	0.030625	0.033320	0.040885
2.500000	0.018464	0.021342	0.027471
3.000000	0.011514	0.014002	0.018779
3.500000	0.007372	0.009366	0.013023
4.000000	0.004823	0.006367	0.009143
4.500000	0.003212	0.004388	0.006488
5.000000	0.002172	0.003061	0.004647
6.000000	0.001032	0.001535	0.002443
7.000000	0.000511	0.000796	0.001321
8.000000	0.000262	0.000425	0.000730
9.000000	0.000138	0.000232	0.000412
10.000000	0.000074	0.000129	0.000237
15.000000	0.000004	0.000009	0.000018
20.000000	0.000000	0.000001	0.000002

Table 3 (Cont'd)

 $u = 0.00$

x	δ_0	δ_1	δ_2
0.	0.886227	0.500000	0.443113
0.010000	0.838745	0.491400	0.438157
0.020000	0.804955	0.483187	0.433284
0.030000	0.776229	0.475284	0.428492
0.040000	0.750737	0.467652	0.423777
0.050000	0.727609	0.460262	0.419138
0.060000	0.706336	0.453093	0.414572
0.070000	0.686581	0.446130	0.410076
0.080000	0.668105	0.439357	0.405648
0.090000	0.650731	0.432764	0.401288
0.100000	0.634321	0.426340	0.396992
0.150000	0.563508	0.396463	0.376437
0.200000	0.506231	0.369765	0.357293
0.250000	0.458329	0.345684	0.339417
0.300000	0.417407	0.323816	0.322688
0.350000	0.381920	0.303853	0.307004
0.400000	0.350797	0.285552	0.292275
0.450000	0.323259	0.268714	0.278424
0.500000	0.298717	0.253176	0.265382
0.600000	0.256889	0.225468	0.241485
0.700000	0.222647	0.201546	0.220162
0.800000	0.194206	0.180746	0.201071
0.900000	0.170310	0.162554	0.183926
1.000000	0.150046	0.146563	0.168487
1.500000	0.083910	0.090026	0.110703
2.000000	0.049876	0.057447	0.074539
2.500000	0.030899	0.037688	0.051148
3.000000	0.019739	0.025264	0.035642
3.500000	0.012916	0.017235	0.025159
4.000000	0.008619	0.011931	0.017957
4.500000	0.005847	0.008363	0.012941
5.000000	0.004022	0.005927	0.009406
6.000000	0.001972	0.003061	0.005081
7.000000	0.001005	0.001631	0.002815
8.000000	0.000528	0.000892	0.001593
9.000000	0.000285	0.000498	0.000918
10.000000	0.000157	0.000284	0.000538
15.000000	0.000010	0.000021	0.000045
20.000000	0.000001	0.000002	0.000005

Table 3 (Cont'd)

 $u = 0.20$

x	δ_0	δ_1	δ_2
0.	1.083591	0.697113	0.681218
0.010000	1.034235	0.686548	0.674301
0.020000	0.998189	0.676391	0.667486
0.030000	0.967134	0.666568	0.660772
0.040000	0.939296	0.657038	0.654154
0.050000	0.913830	0.647774	0.647630
0.060000	0.890237	0.638755	0.641198
0.070000	0.868187	0.629964	0.634854
0.080000	0.847445	0.621387	0.628597
0.090000	0.827837	0.613011	0.622426
0.100000	0.809225	0.604827	0.616337
0.150000	0.727903	0.566471	0.587071
0.200000	0.660927	0.531800	0.559628
0.250000	0.604073	0.500211	0.533840
0.300000	0.554873	0.471265	0.509563
0.350000	0.511717	0.444623	0.486675
0.400000	0.473474	0.420012	0.465067
0.450000	0.439314	0.397208	0.444644
0.500000	0.408602	0.376023	0.425319
0.600000	0.355633	0.337896	0.389667
0.700000	0.311630	0.304599	0.357579
0.800000	0.274596	0.275339	0.328613
0.900000	0.243103	0.249495	0.302398
1.000000	0.216094	0.226569	0.278617
1.500000	0.125570	0.143656	0.187929
2.000000	0.077028	0.094171	0.129477
2.500000	0.049032	0.063262	0.090699
3.000000	0.032085	0.043321	0.064405
3.500000	0.021457	0.030135	0.046262
4.000000	0.014607	0.021241	0.033560
4.500000	0.010093	0.015142	0.024558
5.000000	0.007064	0.010902	0.018110
6.000000	0.003574	0.005796	0.010048
7.000000	0.001873	0.003171	0.005705
8.000000	0.001011	0.001776	0.003303
9.000000	0.000559	0.001015	0.001945
10.000000	0.000315	0.000590	0.001162
15.000000	0.000023	0.000048	0.000105
20.000000	0.000002	0.000005	0.000012

Table 3 (Cont'd)

 $u = 0.40$

x	ε_0	ε_1	ε_2
0.	1.265879	0.932424	1.005909
0.010000	1.217792	0.920027	0.996647
0.020000	1.181663	0.908034	0.987507
0.030000	1.150087	0.896378	0.978486
0.040000	1.121475	0.885023	0.969579
0.050000	1.095069	0.873942	0.960784
0.060000	1.070418	0.863116	0.952099
0.070000	1.047224	0.852528	0.943521
0.080000	1.025273	0.842167	0.935048
0.090000	1.004405	0.832019	0.926677
0.100000	0.984496	0.822076	0.918407
0.150000	0.896361	0.775125	0.878495
0.200000	0.822401	0.732205	0.840827
0.250000	0.758644	0.692716	0.805217
0.300000	0.702734	0.656210	0.771506
0.350000	0.653111	0.622338	0.739552
0.400000	0.608670	0.590813	0.709233
0.450000	0.568586	0.561398	0.680436
0.500000	0.532224	0.533893	0.653061
0.600000	0.468746	0.483938	0.602223
0.700000	0.415222	0.439813	0.556080
0.800000	0.369565	0.400632	0.514095
0.900000	0.330259	0.365688	0.475812
1.000000	0.296165	0.334407	0.440836
1.500000	0.178763	0.218699	0.304985
2.000000	0.113139	0.147199	0.214870
2.500000	0.073985	0.101215	0.153579
3.000000	0.049585	0.070783	0.111085
3.500000	0.033885	0.050194	0.081167
4.000000	0.023529	0.036015	0.059830
4.500000	0.016560	0.026105	0.044444
5.000000	0.011791	0.019091	0.033244
6.000000	0.006155	0.010447	0.018940
7.000000	0.003319	0.005867	0.011018
8.000000	0.001837	0.003366	0.006524
9.000000	0.001040	0.001968	0.003923
10.000000	0.000600	0.001169	0.002391
15.000000	0.000048	0.000104	0.000236
20.000000	0.000005	0.000012	0.000028

Table 3 (Cont'd)

 $u = 0.60$

x	ε_0	ε_1	ε_2
0.	1.421380	1.201666	1.431690
0.010000	1.377242	1.187691	1.419743
0.020000	1.343036	1.174093	1.407935
0.030000	1.312677	1.160817	1.396260
0.040000	1.284852	1.147831	1.384717
0.050000	1.258930	1.135114	1.373303
0.060000	1.234537	1.122648	1.362014
0.070000	1.211423	1.110419	1.350849
0.080000	1.189408	1.098416	1.339805
0.090000	1.168358	1.086628	1.328880
0.100000	1.148166	1.075046	1.318072
0.150000	1.057564	1.019967	1.265715
0.200000	0.980055	0.969073	1.216006
0.250000	0.912174	0.921802	1.168748
0.300000	0.851837	0.877730	1.123772
0.350000	0.797641	0.836516	1.080927
0.400000	0.748579	0.797880	1.040077
0.450000	0.703890	0.761586	1.001100
0.500000	0.662979	0.727429	0.963883
0.600000	0.590681	0.664843	0.894330
0.700000	0.528797	0.608946	0.830692
0.800000	0.475290	0.558805	0.772349
0.900000	0.428652	0.513660	0.718765
1.000000	0.387735	0.472884	0.669472
1.500000	0.242933	0.318674	0.474578
2.000000	0.158558	0.220069	0.341641
2.500000	0.106481	0.154804	0.249003
3.000000	0.073069	0.110510	0.183368
3.500000	0.051013	0.079859	0.136234
4.000000	0.036124	0.058312	0.102001
4.500000	0.025891	0.042964	0.076894
5.000000	0.018751	0.031908	0.058325
6.000000	0.010098	0.017965	0.034104
7.000000	0.005599	0.010352	0.020317
8.000000	0.003181	0.006084	0.012300
9.000000	0.001844	0.003636	0.007551
10.000000	0.001088	0.002205	0.004693
15.000000	0.000095	0.000215	0.000504
20.000000	0.000010	0.000026	0.000065

Table 3 (Cont'd)

 $u = 0.80$

x	g_0	g_1	g_2
0.	1.543896	1.498763	1.970959
0.010000	1.505457	1.483530	1.956048
0.020000	1.474647	1.468633	1.941287
0.030000	1.446850	1.454027	1.926674
0.040000	1.421066	1.439689	1.912206
0.050000	1.396810	1.425601	1.897880
0.060000	1.373795	1.411749	1.883693
0.070000	1.351826	1.398121	1.869644
0.080000	1.330764	1.384709	1.855730
0.090000	1.310505	1.371503	1.841949
0.100000	1.290965	1.358497	1.828299
0.150000	1.202065	1.296227	1.761949
0.200000	1.124507	1.238102	1.698607
0.250000	1.055494	1.183634	1.638078
0.300000	0.993308	1.132440	1.580190
0.350000	0.936776	1.084209	1.524785
0.400000	0.885044	1.038682	1.471723
0.450000	0.837456	0.995636	1.420875
0.500000	0.793495	0.954876	1.372122
0.600000	0.714846	0.879554	1.280466
0.700000	0.646509	0.811564	1.195967
0.800000	0.586621	0.749971	1.117940
0.900000	0.533775	0.694005	1.045785
1.000000	0.486880	0.643018	0.978973
1.500000	0.316326	0.445966	0.710259
2.000000	0.212762	0.315710	0.521988
2.500000	0.146652	0.227035	0.387674
3.000000	0.102996	0.165350	0.290485
3.500000	0.073433	0.121708	0.219334
4.000000	0.053016	0.090402	0.166731
4.500000	0.038685	0.067684	0.127507
5.000000	0.028491	0.051032	0.098040
6.000000	0.015822	0.029545	0.058800
7.000000	0.009020	0.017464	0.035859
8.000000	0.005255	0.010507	0.022185
9.000000	0.003119	0.006418	0.013899
10.000000	0.001881	0.003973	0.008807
15.000000	0.000180	0.000422	0.001028
20.000000	0.000021	0.000054	0.000142

Table 3 (Cont'd)

 $u = 1.00$

x	ε_0	ε_1	ε_2
0.	1.633051	1.816990	2.633516
0.010000	1.600976	1.800830	2.615427
0.020000	1.574328	1.784956	2.597498
0.030000	1.549874	1.769336	2.579727
0.040000	1.526907	1.753953	2.562111
0.050000	1.505085	1.738794	2.544647
0.060000	1.484204	1.723848	2.527334
0.070000	1.464124	1.709107	2.510169
0.080000	1.444747	1.694564	2.493151
0.090000	1.425995	1.680210	2.476278
0.100000	1.407809	1.666042	2.459546
0.150000	1.323914	1.597793	2.377968
0.200000	1.249285	1.533497	2.299701
0.250000	1.181820	1.472746	2.224559
0.300000	1.120206	1.415217	2.152373
0.350000	1.063527	1.360643	2.082988
0.400000	1.011105	1.308794	2.016263
0.450000	0.962413	1.259470	1.952067
0.500000	0.917027	1.212497	1.890277
0.600000	0.834843	1.124993	1.773471
0.700000	0.762376	1.045206	1.665022
0.800000	0.698021	0.972248	1.564202
0.900000	0.640542	0.905373	1.470369
1.000000	0.588963	0.843943	1.382947
1.500000	0.396237	0.601522	1.025575
2.000000	0.274372	0.436061	0.768709
2.500000	0.193962	0.320311	0.581286
3.000000	0.139333	0.237837	0.442884
3.500000	0.101401	0.178213	0.339660
4.000000	0.074603	0.134591	0.262016
4.500000	0.055401	0.102351	0.203180
5.000000	0.041478	0.078314	0.158304
6.000000	0.023740	0.046593	0.097318
7.000000	0.013909	0.028237	0.060718
8.000000	0.008309	0.017384	0.038371
9.000000	0.005047	0.010850	0.024524
10.000000	0.003110	0.006853	0.015834
15.000000	0.000327	0.000795	0.002006
20.000000	0.000042	0.000110	0.000296

Table 3 (Cont'd)

 $u = 1.50$

x	g_0	g_1	g_2
0.	1.742415	2.666322	4.870690
0.010000	1.723922	2.648993	4.844114
0.020000	1.707102	2.631839	4.817710
0.030000	1.691019	2.614849	4.791476
0.040000	1.675465	2.598017	4.765412
0.050000	1.660339	2.581338	4.739516
0.060000	1.645577	2.564809	4.713785
0.070000	1.631138	2.548425	4.688219
0.080000	1.616989	2.532185	4.662816
0.090000	1.603108	2.516085	4.637575
0.100000	1.589475	2.500122	4.612494
0.150000	1.524549	2.422292	4.489447
0.200000	1.464181	2.347591	4.370212
0.250000	1.407616	2.275811	4.254639
0.300000	1.354348	2.206775	4.142586
0.350000	1.304008	2.140327	4.033919
0.400000	1.256306	2.076330	3.928513
0.450000	1.211005	2.014657	3.826247
0.500000	1.167908	1.955193	3.727010
0.600000	1.087672	1.842477	3.537193
0.700000	1.014488	1.737424	3.358259
0.800000	0.947480	1.639374	3.189475
0.900000	0.885936	1.547746	3.030170
1.000000	0.829262	1.462024	2.879729
1.500000	0.603603	1.107369	2.242068
2.000000	0.446754	0.847053	1.756722
2.500000	0.334868	0.653171	1.383992
3.000000	0.253566	0.507115	1.095611
3.500000	0.193639	0.396059	0.871064
4.000000	0.148953	0.310947	0.695242
4.500000	0.115310	0.245272	0.556888
5.000000	0.089770	0.194291	0.447528
6.000000	0.055213	0.123302	0.291598
7.000000	0.034533	0.079291	0.192017
8.000000	0.021909	0.051577	0.127630
9.000000	0.014073	0.033891	0.085547
10.000000	0.009139	0.022472	0.057775
15.000000	0.001200	0.003213	0.008913
20.000000	0.000185	0.000527	0.001552

Table 3 (Cont'd)

 $u = 2.0$

x	ξ_0	ξ_1	ξ_2
0.	1.768308	3.545774	7.975701
0.010000	1.757127	3.528147	7.940332
0.020000	1.746296	3.510630	7.905138
0.030000	1.735650	3.493221	7.870119
0.040000	1.725151	3.475917	7.835274
0.050000	1.714781	3.458717	7.800601
0.060000	1.704528	3.441621	7.766099
0.070000	1.694383	3.424626	7.731768
0.080000	1.684341	3.407733	7.697606
0.090000	1.674397	3.390939	7.663613
0.100000	1.664546	3.374244	7.629787
0.150000	1.616595	3.292225	7.463135
0.200000	1.570619	3.212552	7.300525
0.250000	1.526431	3.135133	7.141842
0.300000	1.483890	3.059882	6.986976
0.350000	1.442885	2.986718	6.835819
0.400000	1.403324	2.915569	6.688270
0.450000	1.365125	2.846363	6.544230
0.500000	1.328216	2.779035	6.403603
0.600000	1.258021	2.649762	6.132221
0.700000	1.192288	2.527282	5.873424
0.800000	1.130637	2.411168	5.626552
0.900000	1.072737	2.301030	5.390991
1.000000	1.018295	2.196506	5.166159
1.500000	0.789776	1.747271	4.184968
2.000000	0.617856	1.397344	3.402391
2.500000	0.486747	1.122640	2.775123
3.000000	0.385724	0.905599	2.270165
3.500000	0.307230	0.733173	1.862105
4.000000	0.245814	0.595531	1.531207
4.500000	0.197469	0.485186	1.262034
5.000000	0.159212	0.396385	1.042438
6.000000	0.104510	0.266570	0.715502
7.000000	0.069388	0.180892	0.494688
8.000000	0.046528	0.123737	0.344273
9.000000	0.031474	0.085252	0.241029
10.000000	0.021459	0.059120	0.169675
15.000000	0.003477	0.010282	0.031443
20.000000	0.000636	0.001987	0.006392

Table 3 (Cont'd)

 $u = 2.50$

x	δ_0	δ_1	δ_2
0.	1.772093	4.431197	11.964040
0.010000	1.764153	4.413516	11.919816
0.020000	1.756284	4.395914	11.875769
0.030000	1.748467	4.378390	11.831898
0.040000	1.740698	4.360944	11.788201
0.050000	1.732975	4.343576	11.744678
0.060000	1.725297	4.326284	11.701329
0.070000	1.717663	4.309070	11.658152
0.080000	1.710071	4.291931	11.615147
0.090000	1.702521	4.274868	11.572313
0.100000	1.695012	4.257881	11.529650
0.150000	1.658069	4.174058	11.318859
0.200000	1.622090	4.092058	11.112214
0.250000	1.587032	4.011834	10.909624
0.300000	1.552860	3.933340	10.711001
0.350000	1.519541	3.856534	10.516261
0.400000	1.487046	3.781372	10.325320
0.450000	1.455349	3.707816	10.138097
0.500000	1.424423	3.635824	9.954513
0.600000	1.364792	3.496388	9.597952
0.700000	1.307975	3.362772	9.255041
0.800000	1.253813	3.234704	8.925213
0.900000	1.202155	3.111926	8.607924
1.000000	1.152866	2.994194	8.302659
1.500000	0.937789	2.473576	6.940193
2.000000	0.766032	2.049205	5.813072
2.500000	0.628002	1.701936	4.878161
3.000000	0.516497	1.416791	4.100800
3.500000	0.426019	1.181942	3.453000
4.000000	0.352318	0.987981	2.912054
4.500000	0.292073	0.827385	2.459466
5.000000	0.242673	0.694104	2.080122
6.000000	0.168557	0.490867	1.493797
7.000000	0.117943	0.349202	1.077969
8.000000	0.083073	0.249763	0.781386
9.000000	0.058861	0.179523	0.568756
10.000000	0.041933	0.129625	0.415590
15.000000	0.008237	0.026965	0.091030
20.000000	0.001770	0.006068	0.021366

Table 3 (Cont'd)

 $u = 5.00$

x	δ_0	δ_1	δ_2
0.	1.772453	8.862266	45.197562
0.020000	1.765227	8.826890	45.020671
0.040000	1.758032	8.791658	44.844485
0.060000	1.750866	8.756569	44.669004
0.080000	1.743730	8.721623	44.494221
0.100000	1.736623	8.686819	44.320138
0.200000	1.701534	8.514917	43.460080
0.300000	1.667169	8.346488	42.617039
0.400000	1.633514	8.181460	41.790669
0.500000	1.600553	8.019763	40.980635
0.600000	1.568273	7.861327	40.186607
0.700000	1.536657	7.706086	39.408263
0.800000	1.505694	7.553974	38.645288
0.900000	1.475367	7.404926	37.897366
1.000000	1.445665	7.258880	37.164201
1.500000	1.306068	6.571550	33.709499
2.000000	1.180221	5.950520	30.581607
2.500000	1.066739	5.389269	27.749024
3.000000	0.964383	4.881928	25.183355
3.500000	0.872038	4.423218	22.858992
4.000000	0.788706	4.008388	20.752825
4.500000	0.713489	3.633160	18.844005
5.000000	0.645580	3.293682	17.113709
6.000000	0.528864	2.708438	14.122370
7.000000	0.433599	2.228813	11.661677
8.000000	0.355774	1.835435	9.636034
9.000000	0.292141	1.512542	7.967346
10.000000	0.240068	1.247305	6.591758
15.000000	0.090923	0.480326	2.578353
20.000000	0.035006	0.187789	1.022736
25.000000	0.013679	0.074438	0.410926
30.000000	0.005419	0.029884	0.167077
35.000000	0.002174	0.012139	0.068685
40.000000	0.000882	0.004985	0.028529
45.000000	0.000362	0.002068	0.011965
50.000000	0.000150	0.000866	0.005064
60.000000	0.000026	0.000156	0.000930
70.000000	0.000005	0.000029	0.000176
80.000000	0.000001	0.000006	0.000034
90.000000	0.000000	0.000001	0.000007
100.000000	0.000000	0.000000	0.000001
150.000000	0.000000	0.000000	0.000000
200.000000	0.000000	0.000000	0.000000

Table 3 (Cont'd)

 $u = 10.00$

x	g_0	g_1	g_2
0.	1.772453	17.724533	178.131565
0.020000	1.768894	17.689120	177.777435
0.040000	1.765342	17.653777	177.424006
0.060000	1.761797	17.618506	177.071274
0.080000	1.758259	17.583305	176.719259
0.100000	1.754729	17.548176	176.367941
0.200000	1.737182	17.373581	174.621872
0.300000	1.719812	17.200734	172.893169
0.400000	1.702616	17.029614	171.181665
0.500000	1.685593	16.860204	169.487192
0.600000	1.668741	16.692489	167.809568
0.700000	1.652058	16.526451	166.148636
0.800000	1.635544	16.362072	164.504227
0.900000	1.619195	16.199337	162.876165
1.000000	1.603010	16.038228	161.264305
1.500000	1.524493	15.256518	153.442249
2.000000	1.449840	14.513092	146.001406
2.500000	1.378861	13.806066	138.923090
3.000000	1.311374	13.133649	132.189566
3.500000	1.247206	12.494138	125.783970
4.000000	1.186193	11.885917	119.690224
4.500000	1.128179	11.307446	113.893092
5.000000	1.073017	10.757263	108.378063
6.000000	0.970689	9.736269	98.139820
7.000000	0.878164	8.812619	88.873084
8.000000	0.794499	7.976990	80.485248
9.000000	0.718841	7.220954	72.892578
10.000000	0.650421	6.536896	66.019358
15.000000	0.394758	3.977186	40.264552
20.000000	0.239888	2.422741	24.586037
25.000000	0.145954	1.477601	15.030066
30.000000	0.088909	0.902233	9.198833
35.000000	0.054225	0.551549	5.636341
40.000000	0.033110	0.337557	3.457384
45.000000	0.020240	0.206824	2.123133
50.000000	0.012387	0.126864	1.305208
60.000000	0.004655	0.047889	0.494856
70.000000	0.001757	0.018155	0.188407
80.000000	0.000666	0.006911	0.072026
90.000000	0.000254	0.002642	0.027645
100.000000	0.000097	0.001014	0.010652
150.000000	0.000001	0.000009	0.000096
200.000000	0.000000	0.000000	0.000001